

## I B. Tech I Semester Regular Examinations, July/August-2021

## MATHEMATICS-I

(Com. to All Branches)

Time: 3 hours

Max. Marks: 70

**Answer any five Questions one Question from Each Unit**  
**All Questions Carry Equal Marks**

1 a) Examine the convergence of  $\sum \frac{[(n+1)!]^2 x^{n-1}}{n}$ , ( $x > 0$ ) (7M)

b) Find Maclaurin's series expansion of the  $f(x, y) = \sin^2 x$  and hence find the approximate value of  $\sin^2 16^\circ$ . (7M)

Or

2. a) Prove using mean value theorem  $|\sin u - \sin v| \leq |u - v|$ . (7M)

b) Examine the convergence of  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$  ( $x > 0$ ). (7M)

3. a) Solve  $(x + 2y^3) \frac{dy}{dx} = y$ . (7M)

b) Solve  $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$  (7M)

Or

4. a) Find the orthogonal trajectories of  $r^2 = a \sin 2\theta$ . (7M)

b) Solve  $(xysinx + cosxy) ydx + (xysinxy - cosxy) xdy = 0$ . (7M)

5. a) Solve  $(D^3 - D)y = 2x + 1 + 4\cos x + 2e^x$  (7M)

b) In an L-C-R circuit, the charge  $q$  on a plate of a condenser is given by (7M)

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = E \sin pt$$

The circuit is tuned to resonance so that  $q^2 = 1/LC$ . If initially the current  $I$  and the charge  $q$  be zero, show that, for small values of  $R/L$ , the current in the circuit at time  $t$  is given by  $(Et/2L)\sin pt$ .

Or

6. a) Solve  $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$  by the method of variation of parameters. (7M)

b) Solve  $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ . (7M)

7. a) If  $u = \sin^{-1} \left( \frac{x^2 + y^2}{x + y} \right)$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ . (7M)

b) Investigate the maxima and minima, if any, of the function  $f(x) = x^3 y^2 (1 - x - y)$ . (7M)

Or

8. a) Prove that  $u = \frac{x^2 - y^2}{x^2 + y^2}$ ,  $v = \frac{2xy}{x^2 + y^2}$  are functionally dependent and find the relation between them. (7M)

b) Expand  $f(x, y) = e^{x+y}$  in the neighborhood of (1, 1). (7M)

9. a) Evaluate  $\iint_R xy dx dy$  where R is the region bounded by the x-axis, ordinate  $x = 2a$  and the curve  $x^2 = 4ay$ . (7M)

b) By changing the order of integration, evaluate  $\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy$ . (7M)

Or

10 a) Evaluate the following integral  $\int_0^{\pi/2} \int_0^{a \sin \theta} \int_0^{(a^2-r^2)/a} r dr d\theta dz$  (7M)

b) Evaluate  $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2} dy dx$  by changing into polar coordinates. (7M)