

II B. Tech I Semester Regular Examinations, March - 2021
RANDOM VARIABLES AND STOCHASTIC PROCESSES
 (Electronics and Communication Engineering)

Time: 3 hours

Max. Marks: 75

Answer any **FIVE** Questions each Question from each unit
 All Questions carry **Equal** Marks

- 1 a) Define conditional probability distribution function and write the properties [8M]
 b) A random variable X is defined by [7M]

$$X(i) = \begin{cases} -2 & i \leq -2 \\ i & -2 < i \leq 1 \\ 1 & 1 < i \leq 4 \\ 6 & 4 < i \end{cases}$$

Show, by a sketch, the value x into which the values of i are mapped by x.
 What type of random variable is X?

Or

- 2 a) Given that a random variable X has the following possible values, state if X is discrete, continuous or mixed [8M]
 i. $\{-20 < x < -5\}$
 ii. $\{10, 12 < x \leq 14, 15, 17\}$
 iii. $\{-10 \text{ for } s > 2 \text{ and } 5 \text{ for } s \leq 2, \text{ where } 1 < s \leq 6\}$
 iv. $\{4, 3, 1, 1, -2\}$

- b) Suppose height to the bottom of clouds is a Gaussian random variable for which $\mu_x = 4000\text{m}$ and $\sigma_x = 1000\text{m}$. A person bets that cloud height tomorrow will fall in the set $A = \{1000\text{m} < X \leq 3000\text{m}\}$ while a second person bets that height will be satisfied by $B = \{2000\text{m} < X \leq 4200\text{m}\}$. A third person bets they are both correct. Find the probability that each person will win the bet. [7M]

- 3 a) The random variable X has characteristic function $\phi_X(w) = [a/a - jw]^N$ for $a > 0$ and $N = 1, 2, 3, \dots$. Show that $\bar{X} = N/a$, $\bar{X}^2 = N(N+1)/a^2$, and $\sigma_x^2 = N/a^2$. [8M]
 b) Find mean and variance of Gaussian random variable? [7M]

Or

- 4 a) A random variable X is uniformly distributed on the interval $(-5, 15)$. Another random variable $Y = e^{(-X/5)}$ is formed. Find $E[Y]$. [8M]
 b) A Gaussian voltage random variable X has a mean value $\mu_x = 0$ and $\sigma^2_x = 9$. The voltage X is applied to a square-law, full wave diode detector with a transfer characteristics $Y = 5X^2$. Find the mean value of the output voltage Y. [7M]

- 5 a) Random variable X and Y have the joint density [8M]

$$F_{X,Y}(x,y) = \begin{cases} 1/24 & 0 < x < 6 \text{ and } 0 < y < 4 \\ 0 & \text{elsewhere.} \end{cases}$$

What is the expected value of the function $g(X,Y) = (XY)^2$?

- b) Two statistically independent random variable X and Y have mean values $\bar{X} = E[X] = 2$ and $E[Y] = 4$. They have second moments $\bar{X}^2 = E[X^2] = 8$ and $E[Y^2] = 25$. Find i) the mean value ii) the second moment iii) the variance of the random variable $W = 3X - Y$. [7M]

Or

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R19**SET - 1**

- 6 a) For the two random variable X and Y: [8M]

$$F_{X,Y}(x,y) = 0.15\delta(x+1)\delta(y) + 0.1\delta(x)\delta(y) + 0.1\delta(x)\delta(y-2) + 0.4\delta(x-1)\delta(y+2) + 0.2\delta(x-1)\delta(y-1) + 0.5\delta(x-1)\delta(y-3)$$
 Find: i) the correlation, ii) the covariance, iii) the correlation coefficient of X and Y and iv) are X and Y either uncorrelated or orthogonal?
- b) Gaussian random variable X_1 and X_2 for which $\bar{X}_1=2, \sigma_{X_1}^2=9, \bar{X}_2=-1, \sigma_{X_2}^2=4$ and $C_{X_1X_2}=-3$ are transformed to new random variable Y_1 and Y_2 according to $Y_1=-X_1+X_2, Y_2=-2X_1-3X_2$. Find [7M]
 i) $\sigma_{Y_1}^2$ ii) $\sigma_{Y_2}^2$ iii) $C_{Y_1Y_2}$.
- 7 a) Let $X(t)$ be a stationary continuous random process that is differentiable. Denote [8M]
 its time derivative by $\dot{X}(t)$. Show that $E\left[\dot{X}(t)\right] = 0$.
- b) Given the random process by $X(t) = A \cos(w_0t) + B \sin(w_0t)$ [7M]
 Where w_0 is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance, show that $X(t)$ is wide sense stationary but not strictly stationary.
- Or
- 8 a) A random process is defined by $X(t) = A$, where A is a continuous random [8M]
 variable uniformly distributed on (0, 1). Determine the form of the sample functions, classify the process
- b) Define ergodic random proven? Explain with example. [7M]
- 9 a) Drive the Wiener-Khintchine relation. [10M]
- b) What is Mean value of System Response for Random Signal Response of Linear [5M]
 Systems.
- Or
- 10 A Random signal $X(t)$ of PSD of $\frac{N_0}{2}$ is applied on an LTI system having impulse [15M]
 response $h(t)$. If $Y(t)$ is output, find (i) $E[Y^2(t)]$ (ii) $R_{XY}(\tau)$ (iii) $R_{YX}(\tau)$ (iv) $R_{YY}(\tau)$.

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- 1 a) Define Random variable? List out the properties of Distribution Function [8M]
- b) A random variable X is known to be Poisson with  $\lambda=0$  [7M]  
 Plot the density and distribution functions for this random variable.  
 What is the probability of event  $\{0 \leq X \leq 5\}$

Or

- 2 a) Explain Gaussian random variable with neat sketches? [8M]
- b) For the gaussian density function of  $\mu=0$  and  $\sigma_x=1$ , show that [7M]  
 $\int_{-\infty}^{\infty} x f_x(x) dx = \mu$
- 3 a) A random variable X is uniformly distributed on the interval  $(-\pi/2, \pi/2)$ . X [8M]  
 is transformed to the new random variable  $Y = T(X) = a \tan(X)$ , where  $a > 0$ .  
 Find the probability density function of Y.
- b) Show that characteristics function of a random variable having the binomial [7M]  
 density function is  $\Phi(w) = [1 - p + pe^{jw}]^N$ .

Or

- 4 a) A random variable X has  $\bar{X} = -3$ ,  $\bar{X}^2 = 11$  and  $\sigma_x^2 = 2$ . For a new random variable [8M]  
 $Y = 2X - 3$ ,  
 Find: i)  $\bar{Y}$  ii)  $\bar{Y}^2$  iii)  $\sigma_Y^2$ .
- b) For the poisson random variable show that mean and variance is same [7M]
- 5 a) Two gaussian random variables X and Y have variances  $\sigma_X^2 = 9$  and  $\sigma_Y^2 = 4$  [8M]  
 respectively and correlation coefficient  $\rho$ . It is known that a coordinate rotation  
 by angle  $-\pi/8$  results in new random variable  $Y_1$  and  $Y_2$  that are uncorrelated.  
 What is  $\rho$ ?
- b) Two random variables X and Y are defined by  $\bar{X} = 0, \bar{Y} = -1, \bar{X}^2 = 2, \bar{Y}^2 = 4$  and  $R_{XY} = -$  [7M]  
 $2$ . Two random variable W and U are:  $W = 2X + Y, U = -X - 3Y$ . Find  
 $\bar{W}, \bar{U}, \bar{W}^2, \bar{U}^2, R_{WU}, \sigma_W^2, \sigma_U^2$ .

Or

- 6 a) Random variable X and Y have the joint density function [8M]  

$$F_{X,Y}(x,y) = \begin{cases} (x+y)^2/40 & -1 < x < 1 \text{ and } -3 < y < 3 \\ 0 & \text{elsewhere.} \end{cases}$$
 i) find all the second order moments of X and Y  
 ii) what are variances of X and Y.  
 iii) What is the correlation coefficient?
- b) Two gaussian random variables X and Y are variances  $\sigma_X^2 = 4$  and  $\sigma_Y^2 = 9$  [7M]  
 respectively and correlation coefficient  $\rho$ . It is known that a coordinate rotation  
 by angle  $-\pi/4$  results in new random variable  $Y_1$  and  $Y_2$  that are uncorrelated.  
 What is  $\rho$ ?

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- 7 a) Write the properties of Autocorrelation Function of Random Process [6M]  
 b) A Gaussian random process is known to be a WSS process with mean  $\bar{X} = 4$  [9M]  
 and  $R_{XX}(\tau) = 25e^{-3|\tau|}$  where  $\tau = \frac{|t_k - t_i|}{2}$  and  $i, k = 1, 2$ . Find joint Gaussian density function?

Or

- 8 a) What is wide-sense stationary random process and explain with example [8M]  
 b) Define Random Process and classify it. [7M]  
 9 a) A random process had the power density spectrum [8M]

$$S(\omega) = \frac{6\omega^2}{1 + \omega^4}$$

Find the average power in the process

- b) Assume  $X(t)$  is a wide sense stationary process with non zero mean value. show that [7M]

$$S_{xx}(\omega) = 2\pi\bar{X}^2\delta(\omega) + \int_{-\infty}^{\infty} C_{xx}(\tau)e^{-j\omega\tau}d\tau$$

where  $C_{xx}(\tau)$  is the auto covariance function of  $X(t)$ .

Or

- 10 a) Derive the relationship between Cross-Power Density Spectrum and Cross-Correlation Function. [8M]  
 b) Explain Band pass Processes with Properties. [7M]

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1 a) Define Density Function? List out the properties of Density Function [8M]

b) Gaussian random voltages X for which  $a_x = 0$  and  $\sigma_x = 4.2V$  appears across a 100- $\Omega$  resistor with power rating of 0.25W. What is the probability that the voltage will cause an instantaneous power that exceeds the resistor's rating? [7M]

Or

2 a) Define Poisson Random variable? What type of applications it will suitable and give the relationship between Poisson and Binomial Random variable. [8M]

b) For the Gaussian density function of  $\mu=0$  and  $\sigma=1$ , show that [7M]

$$\int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \sigma^2$$

3 a) Explain about Transformation of random variable [8M]

b) For the binomial density function, show that  $E[X] = Np$  and variance =  $Np(1-p)$  [7M]

Or

4 a) Find the mean, variance from moment generation function of uniform distribution? [8M]

b) A random variable X can have values -4, -1,2,3,4 each with probability 1/5. Find: [7M]  
 i) the density function ii) the mean iii) the variance of the random variable  $Y=3X^2$ .

5 a) Define Marginal density function? Find the Marginal density functions of below joint density function. [8M]

$$f_{XY} = \frac{1}{12} u(x)u(y)e^{-x/3}e^{-y/4}$$

b) Two random variables having joint characteristic function [7M]  
 $\phi_{XY}(\omega_1, \omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$ . Find moment's  $m_{10}$ ,  $m_{01}$ ,  $m_{11}$ ?

Or

6 a) Find the density function of  $W=X+Y$ , where the densities of X and Y are assumed to be:  $f_x(x)=4u(x)e^{-4x}$ ;  $f_y(y)=5u(y)e^{-5y}$ . [10M]

b) Joint Sample Space has three elements (1, 1), (2, 2), and (3, 3) with probabilities 0.4, 0.3, 0.3 respectively then draw the Joint Distribution Function diagram [5M]

7 a) Let two random processes X(t) and Y(t) be defined by [8M]

$$X(t) = A \cos \omega_0 t + B \sin \omega_0 t$$

$$Y(t) = B \cos \omega_0 t - A \sin \omega_0 t$$

where A and B are random variables and  $\omega_0$  is a constant. Assume A and B are uncorrelated, zero mean random variables with same variance. Find the cross correlation function  $R_{XY}(t, t+\tau)$ .

b) Write the properties of Cross correlation Function of Random Processes. [7M]

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**R19****SET - 3**

- 8 a) What is strict-sense stationary random process and explain with example. [8M]  
b) What is Cross- Correlation Function and explain its Properties . [7M]
- 9 a) Write the properties of power density spectrum.  
b) If  $X(t)$  is a stationary process, find the power spectrum of  $Y(t) = A_0 + B_0 X(t)$  in [8M]  
term of the power spectrum of  $X(t)$  if  $A_0$  and  $B_0$  are real constants. [7M]
- Or
- 10 a) Explain Band-Limited Processes with Properties. [7M]  
b) If  $X(t)$  is band limited process such that  $S_{xx}(\omega) = 0$ , when  $|\omega| > \sigma$ , prove that [8M]  
 $2[R_{xx}(0) - R_{xx}(\tau)] \leq \sigma^2 \tau^2 R_{xx}(0)$  .

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- 1 a) Define Random variable? Write the conditions for a function to be random variable [6M]  
 b) A random voltage can have any value defined by the set 'S' = {a ≤ s ≤ b}. A quantizer, divides S into 6 equal-sized contiguous subsets and generates random variable X having values {-4, -2, 0, 2, 4, 6}. Each value of X is earned to the midpoint of the subset of 'S' from which it is mapped [9M]  
 i) Sketch the sample space and the mapping to the line that defines the values of X  
 ii) Find a and b?

Or

- 2 a) Explain about Gaussian random variable [8M]  
 b) A Gaussian random variable X has  $a_x = 2$ , and  $\sigma_x = 2$  [7M]  
 i. Find  $P\{X > 1.0\}$   
 ii. Find  $P\{X \leq -1.0\}$
- 3 a) A random variable X has a probability density [8M]  

$$f_x(x) = \begin{cases} (1/2) \cos(x) & -\pi/2 < x < \pi/2 \\ 0 & \text{elsewhere in } x. \end{cases}$$
  
 Find the mean value of the function on  $g(X) = 4X^2$   
 b) Let X be a Poisson random variable then find out its mean and variance [7M]

Or

- 4 a) Find the expected value of the function  $g(X) = X^3$  where X is a random variable defined by the density [8M]  

$$f_x(x) = \left(\frac{1}{2}\right) u(x) \exp(-x/2).$$
  
 b) State and prove Chebchev's inequality? [7M]
- 5 a) For two random variables X and Y [8M]  

$$f_{X,Y}(x,y) = 0.15 \delta(x+1)\delta(y) + 0.1 \delta(x)\delta(y) + 0.1 \delta(x)$$

$$\delta(y-2) + 0.4 \delta(x-1)\delta(y+2) +$$

$$0.2 \delta(x-1)\delta(y-1) + 0.5 \delta(x-1)\delta(y-3).$$

Find the correlation coefficients of X and Y

- b) Gaussian random variables X and Y have first and second order moments  $m_{10} = 1.1$ ,  $m_{20} = 1.16$ ,  $m_{01} = 1.5$ ,  $m_{02} = 2.89$ ,  $R_{XY} = -1.724$  find  $C_{XY}$  and  $\rho$ ? [7M]

Or

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**R19**

**SET - 4**

- 6 a) Define random variables V and W by [8M]

$$V = X + aY$$

$$W = X - aY$$

Where a is real number and X and Y random variables, Determine a in terms of X and Y such V and W are orthogonal?

- b) Gaussian random variable  $X_1$  and  $X_2$  for which  $\bar{X}_1 = 2, \sigma_{X_1}^2 = 9, \bar{X}_2 = -1, \sigma_{X_2}^2 = 4$  and  $C_{X_1 X_2} = -3$  are transformed to new random variable  $Y_1$  and  $Y_2$  according to  $Y_1 = X_1 + X_2, Y_2 = -2X_1 - 3X_2$ . Find [7M]  
 i)  $\bar{X}^2$  ii)  $\bar{X}^2$  iii)  $\rho_{X_1 X_2}$  iv)  $\sigma_{Y_1}^2$

- 7 a) Given that the autocorrelation function for a stationary Ergodic process with no period components is [8M]

$$R_{xx}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$$

Find the mean and variance of process X(t)?

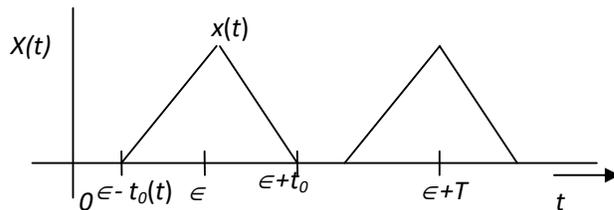
- b) Give the random process by [7M]

$$X(t) = A \cos(w_0 t) + B \sin(w_0 t)$$

Where  $w_0$  is a constant, and A and B are uncorrelated zero mean random variables having different density functions but the same variance, show that X(t) is wide sense stationary but not strictly stationary

Or

- 8 A random process  $X(t)$  has periodic sample functions as show in figure ; where B, T and  $4t_0 \leq T$  are constants but  $\epsilon$  is a random variable uniformly distributed on the interval (0, T). Find first order density function and distribution function of  $X(t)$ . [15M]



- 9 a) Derive the relationship between power spectrum and autocorrelation [8M]

- b) The autocorrelation function of a random process X(t) [7M]

$$R_{xx}(\tau) = 3 + 2 \exp(-4\tau^2)$$

- i. Find the power spectrum of X(t)
- ii. What is the average power in X(t)?

Or

- 10 a) Explain Power Density Spectrum of Response Characteristics of LTI System Response [8M]

- b) Explain Narrowband Processes with Properties. [7M]